Velocity Modulation of Electromagnetic Waves*

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Summary—This paper deals with electromagnetic wave propagation through dielectric media whose propagation constants vary as a function of time.

If the parameters of the medium cannot respond to changes in the electric and magnetic fields of the propagating wave, the fields within such media will be linear. Maxwell's equations are solved for cases in which the scalar permittivity and permeability vary independently with time. When the impedance is constant, an exact solution is obtained. When the impedance varies, a closed form approximation is found since an exact solution is not always possible. The field energy and electromagnetic momentum are derived for a velocity transient and it is seen that, in general, the energy changes and the momentum remains constant.

The frequency deviation that results when a monochromatic wave is passed through a section of dielectric with nonconstant velocity of propagation is discussed in detail. An approximate solution is obtained for the case in which the electrical length of such a section is small; it is found that essentially linear phase modulation occurs. The general solution is found for the case in which the electrical length of section is long and the permittivity of the medium sinusoidally modulated. The optimum length found to give the greatest frequency deviation is shown to be generally impracticable.

It appears that ferroelectric or ferrimagnetic velocity-modulated dielectrics are feasible, at least for low-power modulators.

I. Introduction

THIS paper considers the problem of modulating the velocity of propagation of dielectric media and the effects which arise when electromagnetic waves travel through such media.

A time-varying velocity of propagation implies time-variable permittivity and/or permeability. This immediately suggests that ferroelectrics or ferrimagnetics, respectively under the influence of external electric or magnetic fields, might be suitable means of obtaining velocity-modulated media.

Section II contains the mathematical solution to Maxwell's equations for the general case of independently time-varying μ and ϵ. The special case of a velocity step transient is considered by physical reasoning and the energy densities of the modulated waves are evaluated in Section III. The frequency variation of monochromatic waves passing through a dielectric slab whose velocity of propagation varies homogeneously as a function of time is derived in Section IV. Numerical results based on the published parameters of a particular ferroelectric are given.

* Manuscript received by the PGMTT, July 2, 1957; revised manuscript received, August 5, 1957. This paper is based on a Master of Science thesis submitted to the Electrical Engineering Department of the Massachusetts Institute of Technology, Cambridge, Mass.; June, 1956.

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II. Solution of Maxwell's Equations When the Permittivity and Permeability Vary with Time

The behavior of an electromagnetic wave passing through a dielectric with time-varying velocity of propagation can be predicted if the solution to Maxwell's equations is known when the permittivity and permeability are functions of time.

Assume a charge- and current-free region where μ and ϵ of the medium are functions of time. The wave equations for a TEM mode propagating in the z direction become[1]

$$\frac{\partial^2 E_y}{\partial z^2} = \mu\epsilon\ddot{E}_y + (\dot{\mu}\epsilon + 2\mu\dot{\epsilon})\dot{E}_y + (\mu\ddot{\epsilon} + \dot{\mu}\dot{\epsilon})E_y, \tag{1}$$

and

$$\frac{\partial^2 H_x}{\partial z^2} = \mu\epsilon\ddot{H}_x + (\mu\dot{\epsilon} + 2\dot{\mu}\epsilon)\dot{H}_x + (\ddot{\mu}\epsilon + \dot{\mu}\dot{\epsilon})H_x. \tag{2}$$

The fields may be written in the form

$$E = \frac{W_E}{\epsilon\sqrt{\mu}}\left(Ae^{+j\beta z} + Be^{-j\beta z}\right), \tag{3a}$$

where W_E is given by

$$\ddot{W}_E + \left[\frac{\beta^2}{\mu\epsilon} + \frac{1}{4}\left(\frac{\dot{\mu}}{\mu}\right)^2 - \frac{1}{2}\left(\frac{\ddot{\mu}}{\mu}\right)\right]W_E = 0; \tag{3b}$$

and

$$H = \frac{W_H}{\mu\sqrt{\epsilon}}\left(Ae^{+j\beta z} + Be^{-j\beta z}\right), \tag{4a}$$

where W_H is given by

$$\ddot{W}_H + \left[\frac{\beta^2}{\mu\epsilon} + \frac{1}{4}\left(\frac{\dot{\epsilon}}{\epsilon}\right)^2 - \frac{1}{2}\left(\frac{\ddot{\epsilon}}{\epsilon}\right)\right]W_H = 0. \tag{4b}$$

Special Case

From the form of (3) and (4) it is obvious that W_E will equal W_H only if μ and ϵ are constants or if their ratio is always constant. If $[\mu(t)/e(t) = \eta^2]$ is constant, then it is seen that the ratio of the electric field to the magnetic field will likewise be invariant and so the two fields are in space and time phase. Under these conditions an exact solution of the fields is

[1] F. R. Morgenthaler, "Velocity modulation of electromagnetic waves," M.S. thesis submitted to M.I.T., Cambridge, Mass.; June, 1956.

$$E(z, t) = \eta H(z, t) = \frac{A}{\epsilon} e^{\pm j\beta z} e^{\pm j(\beta/\eta)\int(dt/\epsilon)}, \quad (5)$$

where β takes on eigenvalues subject to the boundary conditions; a series of terms like (5) is the general solution which can be verified by direct substitution. There will be no reflections as long as the impedance of the dielectric remains constant and strictly progressive waves are possible.

It is seen that

$$V(t) = \frac{1}{\sqrt{\mu(t)\epsilon(t)}} = \frac{1}{\eta\epsilon(t)} \cdot \quad (6)$$

The velocity of propagation is given by the same form as when μ and ϵ are constant.

The total phase of the wave given by (5) is

$$\phi = \frac{\beta}{\eta} \int \frac{dt}{\epsilon},$$

and the instantaneous frequency is given by

$$\omega(t) = \frac{d\phi}{dt} = \beta v(t). \quad (7)$$

Eq. (7) indicates that the frequency is simply proportional to the velocity of propagation and this is true if it is remembered that the derivation is based on the assumption that μ and ϵ do not vary with position. This implies that the medium is infinite in extent and, moreover, that any wave now in the dielectric has always been there and has been influenced by any variation in velocity that has occurred since the infinite past. It is appropriate to point out here that the separation of the partial differential equation implies that the space variation of the wave is unaffected by any changes in μ and ϵ. Consider in connection with this that at some point in the distant past a wave train of length L and frequency f_1 was started in the medium and that at that time μ and ϵ were stationary with time. This wave train is characterized by the frequency f_1 and some constant velocity of propagation v_1. It therefore has a wavelength $\lambda_1 = v_1/f_1$. Now suppose that the velocity of propagation suddenly changes to some new value v_2. All portions of the original wave train will be acted upon simultaneously, that is, slowed down or speeded up together. The new wave train will therefore still be of length L and the space waveform will not have changed. This means that the wavelength still has the same value λ_1 but because $v = f\lambda$ it follows that the frequency must have changed to a value $f_2 = v_2/\lambda_1$; hence, $f_2/f_1 = v_2/v_1$. Eq. (7) is merely expressing this fact in general terms. So long as the original wave stays in the medium its frequency will follow the velocity changes of the medium. If a fresh wave enters the dielectric, it will not, of course, become subject to the past history of the medium. For example, if a new wave train of length L and frequency f_1 (as before) enters the dielectric after the velocity has changed from v_1 to v_2, then its frequency will still be f_1 and its wavelength will change to $\lambda_2 = (v_2/v_1)\lambda_1$. The total length will be $(v_2/v_1)L$ instead of L. If the velocity now changes to some new value, the frequency will change accordingly and the wavelength will remain constant.

The exact solution obtained for the special case of constant impedance is illuminating but not very useful since in practice the impedance will not remain constant. It is desirable to solve (3) and (4) for the general case when μ and ϵ vary independently with time. No exact solution is possible and the task remains to find a suitable approximation. A series solution is difficult to interpret physically and so a closed form solution is preferable. Since the equation to be solved is a second-order linear differential equation, the Liouville approximation[2] offers hope and turns out to be entirely suitable.

The fields are then given by

$$E(z, t) \simeq \frac{A}{\sqrt[4]{\mu\epsilon^3}} e^{\pm j\beta z} e^{\pm j\beta\int\left(dt/\sqrt{\mu\epsilon}\right)} \quad (8)$$

and

$$H(z, t) \simeq \frac{A}{\sqrt[4]{\mu^3\epsilon}} e^{\pm j\beta z} e^{\pm j\beta\int\left(dt/\sqrt{\mu\epsilon}\right)}. \quad (9)$$

For slowly varying μ and ϵ, reflections are small and progressive waves are possible. As was shown previously for the special case of constant impedance, the instantaneous frequency is proportional to the velocity of propagation. The physical interpretation given before is also applicable in this situation. The previous remarks concerning eigenvalues of β apply here also so that (8) and (9) are, in general, infinite series.

III. Solution of Step Transient From Physical Reasoning

The fact that the general partial differential (1) and (2) were separable led to the physical interpretation that the space variation of a wave is invariant after it once enters a dielectric having time-varying parameters. The transition across the boundary will certainly cause space distortion but once this has happened no further perturbations of space waveform will occur until the wave leaves the medium. During the journey through the dielectric, all of the individual frequency components of the wave will follow the variations of the velocity of propagation. The physical picture of the phase variations of the electric and magnetic fields is clear: since the wavelength remains constant and the velocity does not, the frequency must change to fulfill the condition $v = f\lambda$. The important point to be realized is that the frequency does not necessarily remain invariant as a wave passes through a series of different dielectrics. That this is so in the usual case is only because the velocity of propagation is not a function of time.

It is desirable to understand why the amplitudes of

[2] S. Schelkunoff, "Applied Mathematics for Engineers and Scientists," D. Van Nostrand Co., Inc., New York, N. Y.; p. 210, 1948.

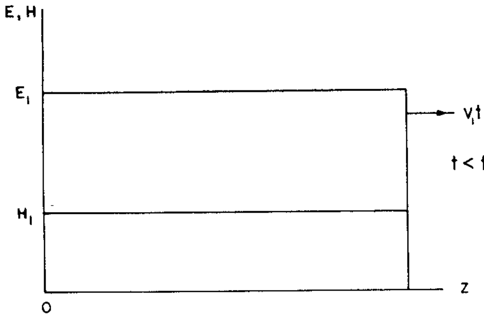

Fig. 1

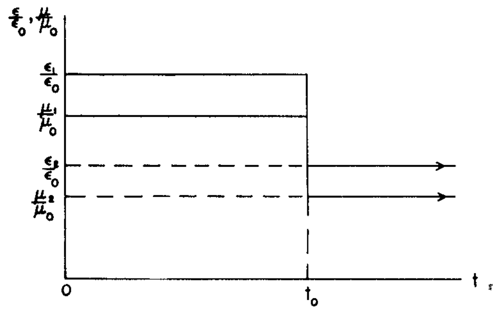

Fig. 2

the electric and magnetic fields vary as they do. Consideration of the instantaneous flux and charge offers a convenient method of obtaining this physical picture. The simple step transient in Fig. 1 offers an easily analyzed example that demonstrates all the relevant principles. The transient electromagnetic wave is propagated through a variable dielectric whose constants are given as functions of time. Both μ and ϵ are assumed to step from their initial values μ_1 and ϵ_1 to μ_2 and ϵ_2, respectively (Fig. 2). The initial velocity of propagation is $v_1 = 1/\sqrt{\mu_1\epsilon_1}$ and the final value is $v_2 = 1/\sqrt{\mu_2\epsilon_2}$. Since the velocity is constant except at the jump, the standard wave equation must apply except at the discontinuity. From the previous discussion it is clear that the space waveform, but not the amplitude of the transient, will be invariant.

At the instant of the jump the total charge Q and the total flux ψ must remain constant. An invariant Q and ψ imply that D and B, respectively, do not change instantaneously.

Before the step $(t < t_0)$

$$B = \mu_1 H_1, \tag{10a}$$

$$D = \epsilon_1 E_1. \tag{10b}$$

After the step $(t > t_0)$

$$B = \mu_2 H_2, \tag{11a}$$

$$D = \epsilon_2 E_2. \tag{11b}$$

The most general form of E_2 and H_2 is for both fields to have a backward as well as a forward traveling-wave component.

$$H_2 = H_2^+ - H_2^-, \tag{12a}$$

$$E_2 = E_2^+ + E_2^-. \tag{12b}$$

The characteristic impedances of the dielectric are

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{E_1}{H_1} \qquad t < t_0, \tag{13a}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{E_2}{H_2} \qquad t > t_0. \tag{13b}$$

The combination of (10a) through (11b) yields

$$B = \mu_1 H_1 = \mu_2(H_2^+ - H_2^-), \tag{14a}$$

$$D = \epsilon_1 E_1 = \epsilon_2(E_2^+ + E_2^-). \tag{14b}$$

The solution for this pair of equations gives

$$E_2^+ = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} + \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\right)E_1 = \frac{\eta_2}{2}\left(\frac{\mu_1}{\mu_2} + \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\right)H_1 = \eta_2 H_2^+, \tag{15a}$$

$$E_2^- = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} - \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\right)E_1 = \frac{\eta_2}{2}\left(\frac{\mu_1}{\mu_2} - \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\right)H_1 = \eta_2 H_2^-. \tag{15b}$$

For the special case when $\eta_1 = \eta_2 = \eta_0$, or equivalently $\mu_1/\mu_2 = \epsilon_1/\epsilon_2$, (15a) and (15b) reduce to

$$E_2^+ = \frac{\epsilon_1}{\epsilon_2}E_1 = \frac{\mu_1}{\mu_2}\eta_0 H_1 = \eta_0 H_2^+, \tag{16a}$$

$$E_2^- = H_2^- = 0. \tag{16b}$$

Under these conditions of constant impedance there is no reflected wave.

For the case where $\mu_1 = \mu_2$, (15a) and (15b) become

$$E_2^+ = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} + \sqrt{\frac{\epsilon_1}{\epsilon_2}}\right)E_1 = \eta_2 H_2^+, \tag{17a}$$

$$E_2^- = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} - \sqrt{\frac{\epsilon_1}{\epsilon_2}}\right)E_1 = \eta_2 H_2^-. \tag{17b}$$

If ϵ_1 and ϵ_2 do not differ greatly, E_2^- will be small compared to E_2^+ and may be neglected without great loss of accuracy. In that event E_2^+ can be approximated by

$$E_2^+ \simeq \frac{\epsilon_1}{\epsilon_2}E_1 \simeq \eta_2 H_2^+. \tag{18}$$

Eq. (18) indicates that the exact solution obtained for the case of constant impedance may be a reasonable approximation when one of the dielectric parameters remains invariant. The Liouville approximation is of slightly different form in that the amplitudes of D and B are not constant.

Energy Density

The uniform step transient of Fig. 1 has a total energy given by

$$U = \frac{1}{2}\int (\epsilon E^2 + \mu H^2)dV, \tag{19}$$

where the integration extends throughout the entire volume. Prior to t_0 the initial energy U_1 is given by

$$U_1 = \tfrac{1}{2}(\epsilon_1 E_1^2 + \mu_1 H_1^2)V. \tag{20}$$

The volume energy density is defined as $u = dU/dV$. For the initial wave

$$u_1 = \epsilon_1 E_1^2 = \mu_1 H_1^2. \tag{21}$$

After the velocity transient $(t > t_0)$, the fields are given by $E_2 = E_2^+ + E_2^-$, and $H_2 = H_2^+ - H_2^-$. The energy density is then

$$u_2 = \frac{1}{2}\epsilon_1 E_1^2\left(\frac{\epsilon_1}{\epsilon_2} + \frac{\mu_1}{\mu_2}\right). \tag{22}$$

The energy gain is defined as u_2/u_1 and given by

$$\frac{u_2}{u_1} = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} + \frac{\mu_1}{\mu_2}\right). \tag{23}$$

The apparent violation of the conservation of energy is reconciled when it is remembered that the difference in energies is needed to do work upon the fields within the dielectric and μ and ϵ are changing. It is apparent that here is a mechanism for changing the energy level of an electromagnetic wave. That the frequency changes as well has already been shown.

Electromagnetic Momentum[3]

The electromagnetic momentum $\vec{g}$ of a field is given by

$$\vec{g} = \frac{1}{v^2}\vec{S}, \tag{24}$$

where v is the velocity of propagation and $\vec{S}$ is Poynting's vector. With reference to the problem just discussed, the original fields are described by (15a) and (15b). It is obvious that

$$g_1 = \epsilon_1\sqrt{\mu_1\epsilon_1}E_1^2. \tag{25}$$

Since $S_2 = (\mu_1\epsilon_1/\mu_2\epsilon_2)E_1^2$, the final value of momentum is given by

$$g_2 = \epsilon_1\sqrt{\mu_1\epsilon_1}E_1^2 = g_1. \tag{26}$$

This important result, derived for the special case of the step transient, is also true for the general case; even though energy may be added or subtracted from the electromagnetic field by varying the velocity of propagation of the medium through which it passes, the electromagnetic momentum of the field is unchanged.

Since the momentum is associated only with the propagated field and not the standing-wave field, it is clear that the Liouville approximation should predict conservation of momentum also. The fact that the exact $\vec{S}_2$ is given by the approximation ensures that this is indeed true.

[3] J. Stratton, "Electromagnetic Theory," McGraw Hill Book Co., Inc., New York, N. Y., pp. 103–104; 1941.

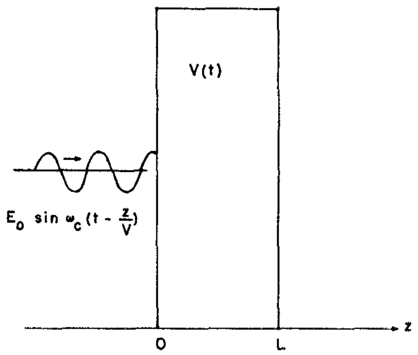

Fig. 3

IV. The Dielectric Modulator

A time-variable dielectric (Fig. 3) extends from $z=0$ to $z=L$ and is assumed homogeneous throughout. A monochromatic wave passes through the section.

It can be shown that the equation for a wavefront propagating with nonconstant velocity[4] is

$$E = E_0 \sin\left[\omega_c\left(t - \int \frac{dz}{v}\right)\right]. \tag{27}$$

The transit time is defined as

$$T = \int \frac{dz}{v}\,. \tag{28}$$

Since the total phase of the wave is given by

$$\phi = \omega_c\left(t - \int \frac{dz}{v}\right), \tag{29}$$

the instantaneous frequency is

$$\omega(t) = \omega_c\left(1 - \frac{d}{dt}T\right). \tag{30}$$

Thin Sections

If the dielectric slab is very thin, the velocity can be approximated as constant for any given wavefront and dependent only upon the time that the wavefront entered the medium during the modulating cycle.[5] Under these conditions

$$T = \int_0^L \frac{dz}{v} \simeq \frac{L}{v(t)}\,. \tag{31}$$

The instantaneous frequency is approximated by

$$\omega(t) \simeq \omega_c\left(1 + \frac{L}{v^2}\frac{dv}{dt}\right). \tag{32}$$

General Solution

In practical situations the approximation (32) is nearly always valid. It is, however, instructive to con-

[4] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., New York, N. Y., pp. 268–269; 1950.

[5] This is analogous to the situation found in klystrons where the velocity of an electron is assumed to be constant while it passes through the narrow accelerating gap but dependent on the time that it entered the gap.

sider the general case when the velocity cannot be assumed constant for the transit interval. The transit time for the entire length L is given by

$$T = \int_0^L \frac{dz}{v} = t_1 - t_0. \tag{33}$$

In this equation, t_0 is the entrance time of a wavefront and t_1 is the exit time of the same wavefront. The integral may be evaluated as follows:

$$\int_{t_0}^{t_1} v(t)dt = \int_0^L d_z = L. \tag{34}$$

Therefore, $f(t_1, t_0) = L$, or $t_0 = g(t_1, L)$ and

$$T = t_1 - g(t_1, L). \tag{35}$$

Sinusoidal Modulation

In the case of monochromatic waves passing through a homogeneous dielectric slab whose permittivity is a sinusoidal function of time, if ϵ is given by

$$\epsilon = K'\epsilon_0(1 + b \sin \omega_m t), \tag{36}$$

and $\mu = \mu_0$, then the velocity of propagation is given by

$$v = \frac{v_0}{\sqrt{1 + b \sin \omega_m t}}; \quad v_0 = \frac{1}{\sqrt{K'\mu_0\epsilon_0}}, \tag{37}$$

where K' is the dielectric constant at the chosen operating point.

If the dielectric is electrically thin, then the approximation (32) can be used. Under these circumstances the instantaneous frequency of the wave emerging from the dielectric is

$$\omega(t) \simeq \omega_c\left[1 - \frac{bL}{2v_0}\frac{\cos \omega_m t}{\sqrt{1 + b \sin \omega_m t}}\right], \tag{38}$$

and the total variation of frequency

$$\Delta\omega(t) \simeq -\frac{bL\omega_m\omega_c \cos \omega_m t}{2v_0\sqrt{1 + b \sin \omega_m t}}. \tag{39}$$

If b is very small, (39) becomes

$$\Delta\omega(t) \simeq -\frac{bL\omega_m\omega_c}{2v_0}\cos \omega_m t, \tag{40}$$

which is the form of linear phase modulation where

$$\omega_d = \frac{b\omega_m\omega_c L}{v_0}, \tag{41}$$

and

$$\theta_d = \frac{\omega_d}{\omega_m} = \frac{b\omega_c L}{v_0}. \tag{42}$$

If L is electrically long so that the velocity cannot be assumed constant over the transit time interval, then the integral (34) must be evaluated. The instantaneous frequency may be shown to be approximately[1]

$$\omega(t_1) \simeq \frac{\sec^2\left(\frac{\omega_m t_1}{2}\right)}{\left[1 + \left\{\frac{2\tan\left(\frac{\omega_m t_1}{2}\right) + b}{\sqrt{4 - b^2}}\right\}^2\right]} \times \frac{\sec^2\left[\tan^{-1}\left\{\frac{2\tan\left(\frac{\omega_m t_1}{2}\right) + b}{\sqrt{4 - b^2}}\right\} - \frac{\omega_m L\sqrt{4 - b^2}}{4v_0}\right]\omega_c}{\left[1 + \left\{\frac{\sqrt{4 - b^2}}{2}\tan\left[\tan^{-1}\left\{\frac{2\tan\left(\frac{\omega_m t_1}{2}\right) + b}{\sqrt{4 - b^2}}\right\} - \frac{\omega_m L\sqrt{4 - b^2}}{4v_0}\right] - \frac{b}{2}\right\}^2\right]}, \tag{43}$$

when ($b^2 \ll 2$). Note that if $(\omega_m L\sqrt{4 - b^2})/4v_0 = K\pi$ ($K = 0, 1, 2, 3 \cdots$), the frequency will be constant and equal to ω_c. This means that if the length L is such as to require an integral number of modulating cycles to elapse before a wavefront passes completely through, then surely all wavefronts will have exactly the same transit time. Since the frequency variation is proportional to the rate of change of transit time, it is obvious that no frequency variation will take place. These null lengths are given by

$$L = \frac{4K\pi v_0}{\omega_m\sqrt{4 - b^2}} \quad (K = 0, 1, 2, 3, \cdots). \tag{44}$$

Since the frequency behavior is periodic, there is no advantage to be gained in making L any longer than some value within the first interval. The optimum length modulator that will result in the greatest frequency variation is evidently somewhere within the interval

$$0 < L_{\text{opt}} < \frac{4v_0}{\omega_m\sqrt{4 - b^2}}.$$

Straightforward maximization shows that the midpoint of the interval yields the optimum length.

It was assumed in the derivation of (43) that $b^2 \ll 2$. It is therefore permissible to write

$$L_{\text{opt}} \simeq \frac{v_0}{2}\tau_m = \frac{c}{2\sqrt{K'}}\tau_m, \tag{45}$$

where c is the free space velocity of light, and τ_m is the modulating period.

The maximum and minimum values of ω for this optimum length may be shown to be

$$\frac{2+b}{2-b}\omega_c \quad \text{and} \quad \frac{2-b}{2+b}\omega_c$$

respectively.

It is obvious that the optimum length modulator is feasible only when the modulating frequency is very high. For low modulating frequencies, a modulator of any physically practical length is well within the assumptions used in deriving (40), and the frequency variation is essentially pure phase modulation.

Numerical Results

Davis and Rubin[6] have published data on $SrTiO_3$—$BaTiO_3$ (27 per cent $SrTiO_3$) ceramics at 3000 mc. Their results show that the relaxation spectrum reported by Powles and Jackson[7] has not been reached and that at room temperature the dielectric constant is approximately 5000, with a loss tangent of 0.1 (with no bias field applied). If a field strength of 10 kv per cm is maintained, the loss decreases slightly and the dielectric constant drops to about 2000.

On the basis of Rubin and Davis's data it is possible to predict the performance of an $SrTiO_3$—$BaTiO_3$ modulator operating with a carrier frequency of 3000 mc. If the ambient temperature is about 25°C, then $\tan \delta \simeq 0.1$ and K' varies between 4000 and 2000 for zero kv per cm and 10 kv per cm, respectively. Assuming a linear change of K' with field strength, it is obvious that a dc bias of 5 kv per cm in series with an ac voltage of magnitude 10 kv per cm peak-to-peak will produce a permittivity given by

$$\epsilon \simeq 3000\epsilon_0(1 + \tfrac{1}{3}\sin \omega_m t). \tag{46}$$

If the values of b and K' in (46) are substituted into (45) the length of the "optimum modulator" is given by

$$L_{\text{opt}} \simeq \frac{3 \times 10^8}{f_m} \text{ cm}. \tag{47}$$

With a loss tangent of 0.1 the loss at 3000 mc is approximately 15 db per cm. If the maximum allowable loss through the modulator is 3 db, then $f_m \geq 15 \times 10^8$ cps. Therefore, as predicted earlier, the optimum modulator is probably not realizable. If a lower value of ω_m is picked and a short L is used so as to keep the loss down, then (41) and (42) apply. The phase deviation at 3000 mc is

$$\theta_d \simeq \frac{11\pi L}{3}, \tag{48}$$

where L is in centimeters.

[6] L. G. Rubin and L. Davis, Jr., "Some dielectric properties of barium-strontium titanate ceramics at 300 megacycles," *J. Appl. Phys.*, vol. 24, pp. 1194–1197; September, 1953.

[7] J. G. Powles and W. Jackson, "The measurement of the dielectric properties of high-permittivity materials at centimeter wavelengths," *Proc. IEE (London)*, vol. 96, part III, pp. 383–389; 1949.

V. Conclusion

The analysis has indicated that essentially linear phase modulation may be expected from a dielectric modulator. The electromagnetic momentum of a wave going through such a modulator is unaffected by the modulation process but the energy level will, in general, be increased. This energy is provided by the modulating source which on the average does work upon the electromagnetic field.

Dielectrics that appear suitable for velocity modulation include ferroelectric ceramics such as the $BaTiO_3$—$SrTiO_3$ compositions. There is a temperature range above the Curie point where these ceramics are still nonlinear and where the losses are substantially reduced. The Curie temperature can be moved over a wide range by altering the concentration of the strontium atoms.

Ferrimagnetic dielectrics are, of course, also applicable for such use. Although these materials are characterized by tensor rather than scalar permeabilities, the main results of the analysis can be applied if effective scalar permeabilities can be determined for the various directions of propagation.

When an electrostatically controlled ferroelectric is used in an FM modulator to vary the capacitance of a tuned circuit, the operating point value of K' determines the operating point capacitance, which in turn determines the carrier frequency. If the value of K' changes because of temperature or other variations, then the carrier will also drift. Observe that in the dielectric velocity modulator that has been discussed, the carrier frequency is not affected by changes in K' and will always be as stable as the generating source. The phase deviation is, of course, sensitive to changes in the dielectric constant, and drift owing to temperature changes may be important.

The maximum modulating rate to which the ferroelectric ceramics will respond is unknown. The extent to which the piezoelectric effect enters the modulation problem is also unknown.

It should be realized that these results assume that the velocity of propagation of the dielectric medium is modulated, *not* by the electromagnetic field passing through it, but by the modulating bias. If this is true, the linear analysis derived is valid; if not, the field relations are nonlinear and much more difficult to solve.

The velocity will not be modulated by the microwave field if the medium cannot respond to microwave frequencies, and this apparently is the case. Even if it is not, the results are applicable if the microwave field is not sufficiently strong to make significant changes in the permittivity.

VI. Acknowledgment

The author wishes to express his gratitude to L. J. Chu of the Massachusetts Institute of Technology for his guidance and helpful criticism.